1. Gas Station Analytics At a certain gas station, 40% of customers use regular gas (event R), 35% use mid-grade (event M), and 25% use premium (event P). Of the customers that use regular gas, 30% their tanks (Event F). Of the customers that use mid-grade gas, 60% their tanks, while of those that use premium, 50% of their tanks. Assume that each customer is drawn independently from the entire pool of customers.

Solution:-

P(R) = .4 , P(M) = .35 , P(P) = .25

P(F/R) = .3 , P(F/M) = .6 , P(F/P) = .5

(a) What is the probability that the next customer will request regular gas and fill the tank?

P(R ⋂ F) = P(F/R).P(R)

= . 4 X .3 = .12 --- (1)

(b) What is the probability that the next customer will fill the tank?

P(F) = P(F/R)P(R) + P(F/M) P(M) + P(F/P) P(P)

P(F) = .3 X .4 + .6X .35 + .5 X .25 = .455 --- (2)

(c) Given that the next customer fills the tank, what is the conditional probability that they use regular gas?

P(R/F) = P(R ⋂ F) / P(F)

= .12/.455 from (1) and (2)

R(R/F ) = .26

In a collection of toys, 1/2 are red, 1/2 are waterproof, and 1/3 are cool. 1/4 are red and waterproof. 1/6 are red and cool. 1/6 are waterproof and cool. 1/6 are neither red, waterproof, nor cool. Each toy has an equal chance of being selected. (a) Draw an area diagram to represent these events. (b) What is the probability of getting a red, waterproof, cool toy? (c) You pull out a toy at random and you observe only the color, noting that it is red. Conditional on just this info

rmation, what is the probability that the toy is not cool? (d) Given that a randomly selected toy is red or waterproof, what is the probability that it is cool?

Probability of red = P(R)

Probability of Waterproof= P(W)

Probability of cool= P(C)

P( R U W U C) = 1-⅙ = 0.833

P( R U W U C)  = P(R) +P(W ) P(C) - P( W U C) - P( R U W) - P( R U  C) + P( R W C)

1 - ⅙ = ½ + ½ + ⅓ - 1/6 - 1/4 - 1/6 +  P( R W C)

P( R W C)  = 0.083

3. On the Overlap of Two Events

Suppose for events A and B, P(A) = 1/2, P(B) = 2/3, but we have no more information about the events.

(a) What are the maximum and minimum possible values for P(A ⋂ B)?

The maximum value for P(A ⋂ B) is ½

That is because P(A ⋂ B) = P(A)\*P(B/A) -- if P(B/A) ==1

Then maximum value for P(A ⋂ B) is ½

The minimum value for P(A ⋂ B) is 0

That is because P(A ⋂ B) = P(A)\*P(B/A) -- if P(B/A) ==0

Then minimum value for P(A ⋂ B) is 0

(b) What are the maximum and minimum possible values for P(A | B)?

Since it is P(A|B) is conditional on B it can range anywhere between 0 and 1

For example consider there are two boxes red box with only red ball and green box with only green ball.

Let R be event of picking a red ball

B be event of picking the red box

Similary G be event of picking the green box

So if event B occurs, the R has probability of 1 .i.e P(R|B) = 1. Since all the balls that can be picked after the picking the red box are red in color

In the contrary if G has occurred all the P(R|G) = 0

Among Berkeley students who have completed w203, 3=4 like

statistics. Among Berkeley students who have not completed w203, only 1=4 like statistics.

Assume that only 1 out of 100 Berkeley students completes w203. Given that a Berkeley

student likes statistics, what is the probability that they have completed w203?