1. Gas Station Analytics At a certain gas station, 40% of customers use regular gas (event R), 35% use mid-grade (event M), and 25% use premium (event P). Of the customers that use regular gas, 30% their tanks (Event F). Of the customers that use mid-grade gas, 60% their tanks, while of those that use premium, 50% of their tanks. Assume that each customer is drawn independently from the entire pool of customers.

Solution:-

P(R) = .4 , P(M) = .35 , P(P) = .25

P(F/R) = .3 , P(F/M) = .6 , P(F/P) = .5

(a) What is the probability that the next customer will request regular gas and fill the tank?

P(R ⋂ F) = P(F/R).P(R)

= . 4 X .3 = .12

(b) What is the probability that the next customer will fill the tank?

P(F) = P(F/R)P(R) + P(F/M) P(M) + P(F/P) P(P)

P(F) = .3 X .4 + .6X .35 + .5 X .25 = .455

(c) Given that the next customer fills the tank, what is the conditional probability that they use regular gas?

P(R/F) = P(R ⋂ F) / P(F)

= .12/.455

3. On the Overlap of Two Events

Suppose for events A and B, P(A) = 1/2, P(B) = 2/3, but we have no more information about the events.

(a) What are the maximum and minimum possible values for P(A ⋂ B)?

The maximum value for P(A ⋂ B) is ½

That is because P(A ⋂ B) = P(A)\*P(B/A) -- if P(B/A) ==1

Then maximum value for P(A ⋂ B) is ½

The minimum value for P(A ⋂ B) is 0

That is because P(A ⋂ B) = P(A)\*P(B/A) -- if P(B/A) ==0

Then minimum value for P(A ⋂ B) is -

(b) What are the maximum and minimum possible values for P(A | B)?

Since it is P(A|B) is conditional on B it can range anywhere between 0 and 1